HYPER VERSION

Received: *April 11, 2000*, Accepted: *May 18, 2000*

RG-flow, gravity and the cosmological constant

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Abstract: We study the low energy effective action S of gravity, induced by integrating out gauge and matter fields, in a general class of Randall-Sundrum type string compactification scenarios with exponential warp factors. Our method combines dimensional reduction with the holographic map between 5-d supergravity and 4-d large-N field theory. Using the classical supergravity approximation, we derive a flow equation of the effective action S that controls its behavior under scale transformations. We find that as a result each extremum of S automatically describes a complete RG trajectory of classical solutions. This implies that, provided the cosmological constant is canceled in the high energy theory, classical flat space backgrounds naturally remain stable under the RG-flow. The mechanism responsible for this stability is that the non-zero vacuum energy generated by possible phase transitions, is absorbed by a dynamical adjustment of the contraction rate of the warp factor.

Keywords: String Duality, Superstring Vacua, Cosmology of Theories beyond the SM.

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JHEP05(2000)034

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1. Introduction

In this paper we will study the low energy effective action S of 4-dimensional gravity, as obtained by integrating out all quantum fluctuations of gauge and matter fields in a general curved background metric $\widehat{g}_{\mu\nu}$. After choosing a particular RG scale μ we can write S as a sum

$$S = S_E + \Gamma\,, \qquad S_E = \frac{1}{\kappa}\int \sqrt{-\widehat{g}}\left(2U + \widehat{R}\right) \tag{1.1}$$

of a local Einstein action and a remaining non-local effective action Γ, induced by the matter and gauge fluctuations at energy scales smaller than μ. Besides on the metric, S also depends on a number of parameters ϕ^I, representing the various masses, expectation values and coupling constants of this gauge and matter theory. The seemingly inevitable presence of the potential term U leads to the well-known problem of the cosmological constant. While one can think of various possible mechanisms or symmetries that could ensure the absence of this term in the high energy theory, for example (extended) supersymmetry, there is no real explanation known for why we do not observe the vacuum energy contributions produced by the various phase transitions that take place at lower energies [1].

In the following we will attempt to shed some new light on this problem. Our approach is directly motivated by the duality between 4-d large-N gauge theory and 5-d supergravity [2], as well as by recent ideas that have appeared in the study of warped string compactification scenarios of the type proposed in [3, 4]. The starting point of our discussion is the holographic formulation of the renormalization group equations in which the RG scale is treated as a physical extra dimension [5, 6, 7].

In [7] it is shown that the standard RG-flow of the effective action in 4-d large-N field theories can be rewritten as a classical Hamilton-Jacobi evolution equation of the 5-d supergravity action. In the following we will study this same idea in the context of warped string compactifications, which can be viewed as generalizations of the holographic duality to 4-d boundary theories with dynamical gravity [4]. We find that the 5-d evolution equations indeed provide a natural extension of the 4-d Einstein equations and the standard RG equations. This extension seems conservative, in the sense that most of the modifications relative to the conventional theories seem negligible at low energies. The most interesting property of our equations, however, is that they appear to be completely self-consistent for any value of the cosmological constant, which in effect decouples from the RG-induced vacuum energy of the matter fluctuations. From the higher-dimensional viewpoint, this decoupling arises due to a dynamical adjustment of the contraction rate of the warp factor, which automatically compensates for any variations in the vacuum energy produced by the matter. As a result our equations are such that, assuming that Λ is cancelled in high energy theory, it will naturally remain zero under the RG-flow.

All our actual calculations will be within the context of classical 5-d supergravity, and thus within the large-N and large-coupling limit of the 4-d field theory. We have tried, however, to formulate our results in such a form that they represent a rather natural and conservative extension of the standard RG framework. We believe therefore that a number of our conclusions will remain valid also when these limits are relaxed. Preliminary results in this direction indeed indicate that there should exist natural extensions of our formalism to other coupling regimes, as well as to finite N. Here, however, we will restrict ourselves to the simplest and most well-controlled case.

Although the motivation and formulation is based on relatively recent insights, several key elements of our proposed scenario have appeared in earlier studies. For example, the idea that the cosmological constant may be cancelled by regarding our

4-d world as embedded in a curved higher-dimensional space was already suggested earlier in [8]. Also the actual low energy mechanism that will be responsible for the cancellation of the vacuum energy term is closely related to earlier proposals for dealing with the cosmological constant. A lucid exposition of these attempts is given in the excellent review of Weinberg [1]. The idea of using the AdS/CFT correspondence in this context was also suggested independently by C. Schmidhuber [9].

This paper is organized as follows. To set the stage, we start with a very brief sketch of the main idea behind our approach, and give a short description of warped compactification scenarios from the point of view of the AdS_5/CFT_4 duality. In the next few sections we then show how, along the lines of [7], the radial evolution equations of the 5-d supergravity action can be reformulated as an RG-flow equation for the 4-d effective action. We also describe how, as a natural by-product of this analysis, the 4-dimensional Einstein equations arise as a low energy reduction of the 5-d equations. We then address the consequences of the RG-flow symmetry of the action for the cosmological constant problem, and show that it implies that classical flat space backgrounds, once they are stable in the UV, also remain stable under the RG-flow. In the last section we draw some conclusions and leave some open questions. Finally, in appendices A and B we present the form of the evolution equations for constant fields, and compute the value of the 4-d Newton's constant for general warped compactifications, generalizing the RS-result [3].

2. Motivation: RG scale as extra dimension

The central element in the holographic correspondence between 5-d supergravity and 4-d gauge theory is the identification of the extra 5-th coordinate, denoted by r, with an RG parameter of the 4-d world [2]. This reinterpretation finds its origin in the following warped form of the 5-d metric:

$$ds_5^2 = dr^2 + a^2(r)\,\widehat{g}_{\mu\nu}\,dx^\mu\,dx^\nu\,. \tag{2.1}$$

Here $\widehat{g}_{\mu\nu}$ represents the background metric as seen by the gauge theory. The prefactor a^2 typically has an exponential dependence on r, growing infinitely large near the asymptotic boundary at $r \to -\infty$ and infinitesimally small at the other end. The 4-d perspective on 5-d physics at some fixed scale thus depends on the radial location, in such a way that shifts in r have the effect of rescalings inside the 4-d world.

In the combined strong-coupling and large-N limit of the gauge theory, the metric (2.1) solves the classical 5-d supergravity equations of motion, as specified via suitable boundary conditions in the asymptotic region $r \to -\infty$. In the field theory, these boundary conditions amount to choosing specific initial data for the RG-flow, as determined by the UV values of the various couplings ϕ^I. The couplings will then propagate in the r-direction as scalar fields of the dual supergravity. The 5-d geometry will thus in general deviate from its most symmetric form; it may for example contain domain wall structures or even naked singularities representing specific RG-flow trajectories towards non-trivial conformal or non-conformal IR quantum field theories [6].

If we take all ϕ^I independent of x^μ, and $g_{\mu\nu} = a^2(r)\,\widehat{g}_{\mu\nu}$ with $\widehat{g}_{\mu\nu}$ some constant curvature metric with $\widehat{R} = k$, the lagrangian of the 5-d gravity and scalar field theory reduces to

$$L = -12\,a^2\dot{a}^2 + k\,a^2 + a^4\left(\frac{1}{2}\,\dot{\phi}_I^2 + V(\phi)\right), \tag{2.2}$$

with $V(\phi)$ some potential on the space of scalar fields. Here we are allowed to take k independent of r, since any possible r-dependence of k can be absorbed in that of the scale factor a. Moreover, according to the holographic postulate, this choice is necessary to ensure that a is identified with the proper RG energy-scale (as measured relative to the scale set by the size of the 4-d space-time).

The above lagrangian describes the classical mechanics of a $(4+1)$-d inflationary cosmology, where the radial direction, instead of the physical time direction, is used as the time variable. This evolution can be quite non trivial, and critically depends on the shape of the potential $V(\phi)$ that drives the radial motion of the scalar fields. However, there is always a conserved quantity due to the invariance under r translations. Moreover, this conserved "energy" must be set equal to zero by means of the Hamilton constraint of the 5-d gravity, which we may write somewhat suggestively as

$$k = -12\,\dot{a}^2 + a^2\left(\frac{1}{2}\,\dot{\phi}_I^2 - V(\phi)\right). \tag{2.3}$$

This relation is preserved under the time-evolution derived from (2.2), so the right-hand side also represents a conserved quantity under the radial evolution. Since r represents the holographic RG parameter, this tells us that the average curvature k of the 4-d space-time is in fact an RG invariant! This somewhat surprising fact is an immediate consequence of the holographic dictionary, which dictates that any change in the curvature of the 4-d slice as a function of r needs to be interpreted as a pure RG rescaling with constant average curvature k, rather than as a physical RG dependence of k.

The above RG-stability of k indeed sounds counter-intuitive, since normally one would expect that any RG evolution will be accompanied with a non-zero increase in vacuum energy, which in the presence of 4-d gravity would curl up the 4-d space-time. In the standard AdS/CFT set-up, however, gravity is decoupled from the boundary theory. From this perspective it is therefore not surprising that the vacuum energy generated by the holographic RG does not directly backreact on the 4-d geometry. However, this is not the complete story. From looking at the explicit terms on the right-hand side of (2.3), we can in fact identify a specific dynamical mechanism that is responsible for this stability, namely that any increase in the vacuum energy produced by the RG-flow of the fields ϕ^I automatically gets compensated by a corresponding increase of the contraction rate of the warp factor.[1]

In the remainder of this paper, we will explore to what extend this idea can be used to shed new light on the cosmological constant problem. For this, the two-main questions that need to be addressed are

(i) Can the holographic correspondence be carried over to a situation *with* dynamical 4-d gravity? And if so

(ii) What are the specific modifications, relative to the standard rules of 4-d effective field theory, that arise from this 5-d perspective on 4-d gravity and the renormalization group?

We start with the first question.

3. Warped compactification

In the standard AdS/CFT set-up, the boundary theory does not contain from 4-d gravity since the AdS-space is taken to be non compact. Hence 5-dimensional graviton modes that extend all the way to the UV boundary $r \to -\infty$ are not normalizable. However, as first pointed out by Randall and Sundrum [3], this situation changes as soon as one introduces a physical brane-like structure (a "Planck brane") that in effect removes this infinite UV region, thereby truncating the r-region to a semi-infinite range. In this case there will exist normalizable fluctuations of the 5-d metric ds^2 that propagate and couple as 4-d graviton modes to the boundary field theory [3]. These graviton modes correspond to fluctuations of the 5-d metric that preserve the warped form (2.1), but with $\widehat{g}_{\mu\nu}$ replaced by a general fluctuating 4-metric. The resulting strength of the 4-d gravity force as a function of the 5-th coordinate r is in complete accordance with the holographic identification of the warp factor a with the RG scale[2] [4].

This mechanism for including a dynamical graviton naturally arises in a general class of string compactifications based on type-IIB orientifolds and/or F-theory [4]. In this case the 6-d internal manifold K_6 can carry via its topology an effective D3-brane charge, which may be compensated by an appropriate number of explicit D3-brane insertions [12]. These D3-branes wrap the 4-d uncompactified world, and are localized as point-like objects inside the K_6. Upon taking into account gravita-

[1] A similar mechanism was also implicit in the paper by Rubakon abd Shoposhnikov [8], and independently suggested within the present context by C. Schmidhuber [9].

[2] An early suggestion that the RS-scenario [3] might be given a holographic interpretation was made by J. Maldacena [11].

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tional backreaction, this typically leads to a warped space-time geometry of the form

$$ds^2 = a^2(r)\, \widehat{g}_{\mu\nu}\, dx^\mu\, dx^\nu + h_{mn}(r)\, dr^m\, dr^n\,. \tag{3.1}$$

Here r^m and h_{mn} denote the coordinates and metric on K_6.

The most extreme type of warped compactification arises when a relatively large number N of D3-branes coalesce inside a small subregion inside the K_6 manifold [4]. In this case, we may visualize the total 10-dimensional target space Σ_{10} as obtained by gluing together two parts

$$\Sigma_{10} = \Sigma_{\rm UV} \cup \Sigma_{\rm IR}\,, \tag{3.2}$$

where $\Sigma_{\rm IR}$ describes the near-horizon geometry close to the location of the N D3-branes, and $\Sigma_{\rm UV}$ the remaining part of the target space. In the near horizon region $\Sigma_{\rm IR}$, it is natural to use the warp factor a^2 to isolate one of the r directions to be like the extra 5-th coordinate r in (2.1). By choosing the remaining 5 coordinates y^m judiciously, the metric (3.1) can then be recast in the form

$$ds^2 = dr^2 + a^2(r)\, \widehat{g}_{\mu\nu}\, dx^\mu\, dx^\nu + \widehat{h}_{mn}(r,y)\, dy^m\, dy^n\,. \tag{3.3}$$

In this way, we have represented the total 10-d geometry as a r-trajectory of 9-d geometries, where each 9-geometry is the product of 4-d space-time with metric $\widehat{g}_{\mu\nu}$ and an internal 5-manifold K_5 described by $\widehat{h}_{mn}$. The near-horizon region $\Sigma_{\rm IR}$ thus typically looks like

$$\Sigma_{\rm IR} = M_5 \times K_5\,, \tag{3.4}$$

where M_5 is a negatively curved 5-d space with metric of the warped form (2.1). The 5-d space M_5 has a boundary $\partial M_5 = R^4$ located at a *finite* radial distance $r = r_0$, being the place $\Sigma_{\rm IR}$ gets glued onto $\Sigma_{\rm UV}$. The warp-factor a in this region typically traverses a large range of values, approaching zero near the D3-branes while attaining a finite maximum value at the boundary $r = r_0$. The second submanifold $\Sigma_{\rm UV}$ looks like the original K_6 with a six-ball B_6 with boundary $\partial B_6 = K_5$ cut out

$$\Sigma_{\rm UV} = R^4 \times (K_6 - B_6)\,. \tag{3.5}$$

Both submanifolds have the same boundary, equal to $R^4 \times K_5$, so that we can indeed glue them together into one single geodesically complete 10-dimensional manifold Σ_{10}.

The low energy effective field theory of this type of compactifications is a 4-dimensional gauge theory with rank equal to N, the number of D3-branes. By construction, we may identify this field theory with the holographic dual to the IIB-string theory, or supergravity if N is large enough, in the near-horizon region $\Sigma_{\rm IR}$. An important difference with the standard AdS/CFT set-up, however, is that at high energies the 4-d theory is augmented to a full-fledged compactified string theory. So in particular it also contains a dynamical 4-d graviton, that behaves exactly as the

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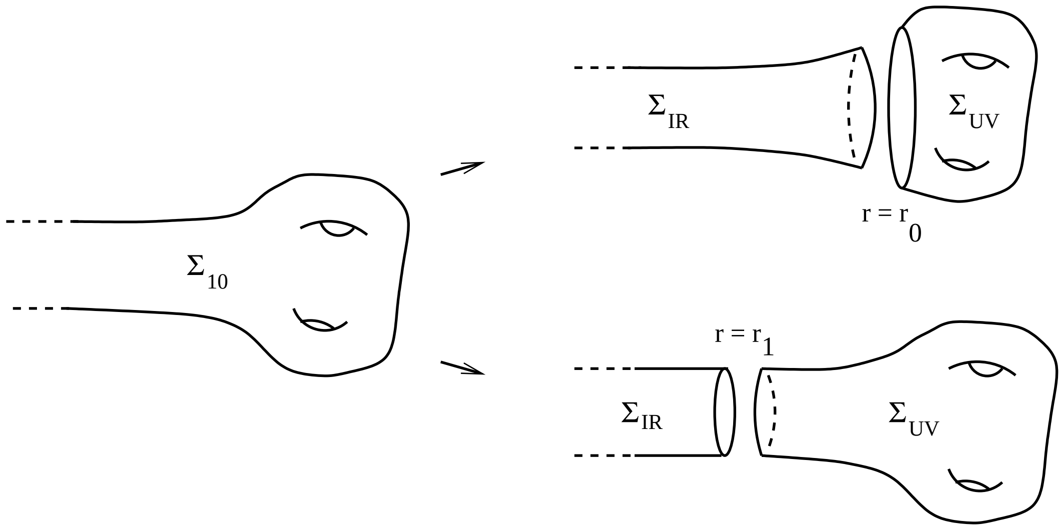

Figure 1: The total warped compactification manifold Σ_{10} can be split up into two submanifolds $\Sigma_{\rm UV}$ and $\Sigma_{\rm IR}$, separated by cutting along a radial location $r = r_0$ close to where the near-horizon tube opens up, or at some location $r = r_1$ farther inside the tube. The corresponding split of the supergravity action S has a holographic interpretation of dividing the 4-d effective action into a high and low energy contribution $S_{\rm UV}$ and $S_{\rm IR}$, separated by a cut-off scale set by r_0 or r_1, respectively. The process of moving the r location from r_0 to r_1 corresponds to performing an RG transformation in the field theory.

bound state graviton in the RS world-brane scenario [3]. Notice, however, that in our case all fields are smooth continuous functions of r, so that, in contrast with the RS set-up, there is no sharply localized brane at any finite value of the warp factor.

4. Splitting the effective action

We would now like to use this holographic perspective to get new insight into the structure of the low energy effective action S of 4-d gravity. To this end we will need to combine the standard techniques of dimensional and low energy reduction with elements of the holographic map between 5-d and 4-d physics. Besides on the 4-d metric, S also depends on various couplings ϕ^I; all are allowed to vary locally with space and time, so that S encodes information about expectation values of local gauge invariant operators $\mathcal{O}_I$ as well as of the stress-energy tensor $T_{\mu\nu}$ of the matter theory. In addition, the metric and couplings ϕ^I now also represent true dynamical degrees of freedom, whose low energy equations of motion are prescribed by S.

Now imagine that we can divide up the total low energy effective action S into a high and low energy contribution, separated by some given RG scale r_0, as

$$S(\phi, g) = S_{\rm UV}(\phi, g) + S_{\rm IR}(\phi, g)\,. \tag{4.1}$$

Here $S_{\rm UV}$ represents the UV part of the effective action, obtained from the original high energy action prescribed by the specific string compactification, by integrating out all degrees of freedom with energy larger than the IR cut-off scale set by r_0, and $S_{\rm IR}$ is the remaining contribution to S from all lower energy degrees of freedom. $S_{\rm UV}$ therefore in essence describes the Einstein action of 4-dimensional gravity, coupled to the scalar fields ϕ^I. The values of the couplings in $S_{\rm UV}$, such as the Newton and cosmological constant, are determined by their initial values as derived from the Kaluza-Klein reduction, corrected by the effects of quantum fluctuations down to the given cut-off scale. $S_{\rm IR}$, on the other hand, can best be thought of as the non-local quantum effective action of the gauge and matter system with UV cut-off set by r_0.

Although the above split can be considered in a general context, it is instructive to view it as the holographic projection inside the boundary theory of the geometric split (3.2) of the space-time manifold in the warped string compactification scenario discussed above. So let us for now assume that the 4-d field theory has a sufficiently strong coupling and large-enough gauge group, so that the dual supergravity system on the warped compactification manifold (3.1) is well approximated by its classical field equations. In this case we can apply the standard AdS/CFT dictionary, and identify the 5-d scalar fields ϕ^I at given radial position r with the matter couplings at the corresponding scale. In particular we can identify the low-energy contribution $S_{\rm IR}$ with the part of the classical supergravity action coming the integral over the near-horizon region $\Sigma_{\rm IR}$

$$S_{\rm IR}(\phi, g) = \int_{\Sigma_{\rm IR}} \mathcal{L}_{\rm sugra}\,. \tag{4.2}$$

Here the integrand is evaluated on a global classical solution, with given boundary values (ϕ, g) and satisfying appropriate asymptotic conditions near the D3-branes. According to the standard AdS/CFT dictionary, this non-local action $S_{\rm IR}$ indeed encodes the information of the gauge theory correlators, defined with a finite UV cut-off.

Now if we in addition assume that the boundary fields vary sufficiently slowly along the non-compact space-time directions, and that the K_6 is large enough compared to the string and Planck scale, then we can similarly represent the high-energy action $S_{\rm UV}$ by the integral over the remaining subspace $\Sigma_{\rm UV}$

$$S_{\rm UV}(\phi, g) = \int_{\Sigma_{\rm UV}} \mathcal{L}_{\rm sugra}\,. \tag{4.3}$$

Again, this integral is to be evaluated for an everywhere regular, classical solution with boundary values (ϕ, g). We can think of this action $S_{\rm UV}$ as obtained from a somewhat modified Kaluza-Klein dimensional reduction of the 10-d supergravity action over the submanifold $K_6 - B_6$. As will become more evident later, the fact that in contrast to the usual KK reduction, the integral runs over an internal manifold with boundary is a reflection of the RG-dependence of the fields on the IR cut-off of $S_{\rm UV}$.

Indeed, since the coordinate location r_0 separating $\Sigma_{\rm UV}$ and $\Sigma_{\rm IR}$ is an adjustable parameter, we are free to consider the evolution of $S_{\rm UV}$ and $S_{\rm IR}$ under variations of this location r_0. In principle we can move r_0 over the whole accessible range of scales. The precise relation between such shifts and actual physical RG scale transformations is dictated by the shape of 4-d part of the 5-d metric (2.1)

$$g_{\mu\nu} = a^2(r)\, \widehat{g}_{\mu\nu} \,. \tag{4.4}$$

The holographic interpretation of this 4-metric $g_{\mu\nu}$ is that it measures distances in units of the RG scale, whereas $\widehat{g}_{\mu\nu}$ is defined relative to some *fixed* length unit such as the fundamental string length. Hence, once we know the shape of the warp factor a as a function of r, we have a well prescribed relation between r-shifts and 4-d RG-scale transformations, since both are now directly linked with constant Weyl-rescalings of $g_{\mu\nu}$. Unless explicitly stated otherwise, $g_{\mu\nu}$ will from now on denote this RG scale dependent metric.

Another lesson we learn from the dual supergravity description is that, in case the couplings ϕ^I vary locally with space and time, we should anticipate that $g_{\mu\nu}$ may change under the r flow in other ways than just simple rescalings. General classical trajectories in 5-d supergravity may describe parameter families of 4-d fields that, as r varies, can change their local shape. Via holography, this means that the 4-d field configuration acquires a non-trivial dependence on the RG scale at which one is looking. Hence rather than fixed RG-independent 4-d field configurations, we are thus lead to consider the notion of RG-trajectories of 4-d backgrounds.

Finally, it is natural to ask whether our definition of the integrals in (4.2) and (4.3) in fact needs to be supplemented with a specific prescription for adding a boundary term or not. Obviously, such a boundary term would automatically cancel in the total sum action S, and should not affect the physics. Our use of the division (4.1) is to set-up a Hamilton-Jacobi formulation of the radial evolution equations of the 5-d supergravity. In this context, it seems most natural to add no boundary term at all.

5. Radial evolution of the effective action

Classical trajectories are selected by requiring that the field configurations (ϕ, g) on the boundary in between $\Sigma_{\rm UV}$ and $\Sigma_{\rm IR}$ must solve the equation of motion of the total action S,

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0\,, \qquad \frac{\delta S}{\delta \phi^I} = 0\,. \tag{5.1}$$

The geometrical meaning of these equations is that they ensure that the UV part of the RG trajectory ending at (ϕ, g) joins smoothly onto the IR part of the trajectory. To see this, it is useful to think about r as a time direction, and recall the standard result in classical field theory that for actions quadratic in time derivatives, there is a linear relation between the field velocities and the variational derivatives of the classical action evaluated at r. Explicitly, in the regime where we can trust the 5-d supergravity, the flow velocities $\dot{g}_{+}^{\mu\nu}$ and $\dot{\phi}_{+}^{I}$ coming from the UV are obtained from $S_{\rm UV}$ via[3]

$$\frac{1}{\sqrt{-g}}\frac{\delta S_{\rm UV}}{\delta g^{\mu\nu}} = \frac{1}{2}\Big(\dot{g}_{\mu\nu} - \dot{g}_{\lambda}^{\lambda} g_{\mu\nu}\Big)_{+}\,, \tag{5.2}$$

$$\frac{1}{\sqrt{-g}}\frac{\delta S_{\rm UV}}{\delta \phi^{I}} = \dot{\phi}_{I,+}\,. \tag{5.3}$$

There exists an identical relation, but with reversed sign, between $S_{\rm IR}$ and the flow velocities $\dot{g}_{-}^{\mu\nu}$ and $\dot{\phi}_{-}^{I}$ coming from the IR. Using that $S = S_{\rm UV} + S_{\rm IR}$, it is easy to see that the continuity condition on the fields and their r-derivatives implies that (ϕ, g) must extremize the low energy action S. Though seemingly self-evident, this will turn out to be a useful observation. In contrast, for instance, it is *not* necessary for classical field configurations to extremize the high or low energy effective actions $S_{\rm UV}$ or $S_{\rm IR}$.

It will be of importance for the following that each classical field configuration lies on one unique RG trajectory. In the 4-d field theory context, this is because the RG-flow is prescribed via a first order differential equation, expressing $\dot{g}_{\mu\nu}$ and $\dot{\phi}^{I}$ as certain given functions of $g_{\mu\nu}$ and ϕ^{I}. Hence once we know the initial position, the flow equation uniquely specifies the trajectory. In the supergravity, on the other hand, this uniqueness is less self-evident, since its equation of motion are second-order differential equations in r. Still the same result holds. The reason is that we are interested in globally well-defined classical trajectories. Equation (4.3) in principle gives $S_{\rm UV}$ as a unique functional of the fields, since the sub-space $\Sigma_{\rm UV}$ has no other boundaries except at the junction with $\Sigma_{\rm IR}$ at r_0. Hence one does not have the freedom to arbitrarily choose both the values and velocities of the fields at its boundary. Rather, it is reasonable to assume that the classical configuration inside $\Sigma_{\rm UV}$ is uniquely determined by just the boundary values of the fields.

The 5-d classical supergravity equation of motion, that prescribes the radial evolution of all quantities, can be most conveniently cast in the form of a Hamilton constraint

$$\frac{1}{4}\Big(\dot{g}^{\mu\nu}\,\dot{g}_{\mu\nu} - \dot{g}_{\mu}^{\mu}\,\dot{g}_{\nu}^{\nu}\Big) + \frac{1}{2}\,\dot{\phi}_{I}^{2} + \mathcal{L}(\phi, g) = 0\,, \tag{5.4}$$

where

$$\mathcal{L}(\phi, g) = -\frac{1}{2}\left(\partial_{\mu}\phi_{I}\right)^{2} + R + V(\phi) \tag{5.5}$$

[3]Throughout this paper we work in the 5-d Einstein frame, using the 5-d Planck length as length unit. In addition we choose the "temporal" gauge $g_{rr} = 1$ and $g_{r\mu} = 0$. Finally, we assume that the metric on the space of couplings ϕ^{I} is flat, so that we can choose it to be δ^{IJ}.

denotes the 4-d part of the 5-d local lagrangian density. Upon inserting the relation (5.2) and the similar relation for $S_{\rm IR}$, this constraint (5.4) takes the form of two functional identities for $S_{\rm UV}$ and $S_{\rm IR}$, which are the familiar Hamilton-Jacobi constraints of the canonical formalism of gravity. Introducing the notation

$$\{S,S\} \equiv \frac{1}{\sqrt{-g}}\left(\frac{1}{3}\left(g^{\mu\nu}\frac{\delta S}{\delta g^{\mu\nu}}\right)^2 - \left(\frac{\delta S}{\delta g^{\mu\nu}}\right)^2 - \frac{1}{2}\left(\frac{\delta S}{\delta \phi^I}\right)^2\right) \tag{5.6}$$

we can write these two equations as

$$\{S_{\rm UV}, S_{\rm UV}\} = \sqrt{-g}\,\mathcal{L} = \{S_{\rm IR}, S_{\rm IR}\}\,. \tag{5.7}$$

Note that the bracket $\{S,S\}$ defines a local density, depending on the point x at which the variational derivatives in (5.6) are taken. Thus the Hamilton-Jacobi equations (5.7) are local constraints. It is further important to note that they are not conditions on the *value* of the UV and IR actions $S_{\rm UV}$ and $S_{\rm IR}$ but rather functional identities that constrain their functional form.

6. RG-flow in terms of beta functions

Let us imagine that we know the high energy action $S_{\rm UV}$ down to some given scale r. We can then use the first of the Hamilton-Jacobi equations (5.7) to integrate further down towards lower scales, by systematically adding the infinitesimal contributions to $S_{\rm UV}$ from each integration step. This procedure is of course inspired by the Wilsonian approach to the renormalization group. To make this RG interpretation more transparent, we express the UV-flow velocities as

$$\dot{\phi}^I_+ = \gamma\,\beta^I\,, \tag{6.1}$$

$$\dot{g}^{\mu\nu}_+ = 2\,\gamma g^{\mu\nu} + \gamma\,\beta^{\mu\nu}\,, \tag{6.2}$$

where $\beta_{\mu\nu}$ is defined to be traceless: $\beta^\mu{}_\mu = 0$. Once we know the explicit form of $S_{\rm UV}$ we can use eq. (5.2) to determine the function γ and the "beta-functions" $\beta_{\mu\nu}$ and β^I as given functionals of the fields. The factor γ represents the flow rate of the conformal factor of the metric, and appears as a common prefactor to ensure that the functions $\beta_{\mu\nu}$ and β^I describe the adjustment of the metric and couplings induced by variations of the physical scale defined by $g_{\mu\nu}$.

It is perhaps helpful to notice the obvious similarity between eqs. (6.1)–(6.2) and the standard kinematic relations in special relativity between the 4-velocity $\dot{x}^\mu$, as measured with respect to the proper time, and the usual 3-velocities $\beta^i = dx^i/dt$ defined relative to the coordinate time $t = x^0$. In this analogy, the extra coordinate r plays the role of the proper time, whereas the scale factor a is the analog of t. The analogy goes quite a bit further, since using the definition (6.1)–(6.2), we can now rewrite the Hamilton-Jacobi constraint for $S_{\rm UV}$ as the following identity for the beta-functions:

$$\gamma^2 \left(1 - \frac{1}{48}\,\beta_{\mu\nu}^2 - \frac{1}{24}\,\beta_I^2\right) = \frac{1}{12}\,\mathcal{L}\,, \tag{6.3}$$

which we recognize as the analog of the familiar relation $\gamma^2(1-\beta^2) = 1$ of special relativity. From this viewpoint it is quite tempting to speculate that for sensible RG-flows, the lagrangian $\mathcal{L}$ on the right-hand side will always be positive, in which case the beta-functions will have a maximum norm squared. Indeed, there are various indications that the standard RG-equations of 4-d QFT follow from the 5-d evolution equations via the direct analog of the non-relatistic limit $\beta^I \ll 1$, see also [7].

From combining the two equations. (5.7) we can in addition derive the following non-linear flow equation for the total low energy effective action S:

$$\gamma \left(2\, g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}} + \beta^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}} + \beta^I \frac{\delta}{\delta \phi^I}\right) S = \{S, S\}\,. \tag{6.4}$$

The physical content of this relation is quite meaningful. The left-hand side takes the form of a standard RG operation, except that it contains functional derivatives, so that it represents a *local* rather than global scale variation. From the perspective of the 4-d field theory, the right-hand side can be thought of as an "anomalous" quantum correction; indeed, as shown in [7], it contains among others the standard Weyl anomaly term. Notice, however, that this right-hand side contribution vanishes for on-shell field configurations that extremize S.

Equation (6.4) tells us that S is invariant under infinitesimal local field variations of the form

$$\delta\phi_I = \epsilon \left(\gamma\, \beta_I + \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \phi^I}\right), \tag{6.5}$$

$$\delta g_{\mu\nu} = \epsilon \left(\gamma\,(-2 g_{\mu\nu} + \beta_{\mu\nu}) + \frac{1}{\sqrt{-g}} \left(\frac{\delta S}{\delta g^{\mu\nu}} - \frac{1}{3}\, g_{\mu\nu}\, g^{\lambda\sigma} \frac{\delta S}{\delta g^{\lambda\sigma}}\right)\right) \tag{6.6}$$

with ϵ some infinitesimal but otherwise arbitrarily varying function of x^μ. This invariance in effect trivializes the dynamics of one local field degree of freedom. When we restrict to the space of classical solutions of S the right-hand sides of (6.5) and (6.6) reduce to the pure RG-flow relations defined in (6.1)–(6.2), which shows that the latter transformation act as a symmetry within the space of extrema to S, i.e. within the space of stable field configurations. This fact will play a central role in the following.

One can notice a close resemblance between eqs. (6.4) and (6.5) and the Polchinski equations for the "exact" renormalization group [13], that describe the RG-scale dependence of a wilsonian action of the fields (ϕ, g) on the cut-off length of their propagators. This similarity perhaps looks surprising since in our case the RG-flow is supposed to be generated by integrating out the dual gauge and matter loops, instead of gravitational degrees of freedom. However, once we realize that ϕ and $g_{\mu\nu}$ are in fact closed string modes and the gauge and matter particles open string modes, it becomes clear that the explanation of this correspondence should follow from the dual equivalence between open string loops and closed string propagators. Indeed, a Polchinski-type UV cut-off that regulates the proper length of a closed string propagator amounts in the dual channel to an IR cut-off on the maximal proper length of the open string loop. It seems that this closed/open string duality lies at the heart of the holographic interplay between gravity — 5-dimensional *as well as* 4-dimensional — and the RG-flow induced by gauge and matter loops. This perspective on the open string RG-flow, as well as its relation with the BV formulation of closed string field theory, will be worked out in more detail in a future publication [14].

7. 4-d Einstein equations

In the following we will study the consequences of these 5-d evolution equations for the case that all fields are approximately space-time independent. In this regime we may truncate the action S_{UV} to the leading order terms in its derivative expansion, and keep essentially only the Einstein part of the action

$$\begin{aligned} S_{\mathrm{UV}}(\phi, g) = \int \sqrt{-\widehat{g}}\left(U(\phi,a) + \Phi(\phi,a)\widehat{R} - \frac{1}{2}\,\partial^{\mu}\phi^{I} M_{IJ}(\phi,a)\,\partial_{\mu}\phi^{J}\right) \equiv \\ \equiv S_E(\phi, g)\,. \end{aligned} \tag{7.1}$$

Here on the right-hand side we have reintroduced the physical metric $\widehat{g}_{\mu\nu}$ defined by extracting the overall RG scale a from $g_{\mu\nu}$, as in eq. (4.4). U represents the vacuum energy contribution from all quantum fluctuations with energies above the scale set by a, and Φ the inverse "Newton constant" at this scale. M_{IJ} denotes some metric on the space of scalar fields. The explicit a-dependence in (7.1) reflects the presence of a fundamental length scale in the problem, given by the string scale. In the holographic parametrization this scale is located at some given value of the conformal factor, which we may set at $a = 1$. So if μ denotes the RG energy scale, a represents the ratio

$$a = \frac{\mu}{M_s}\,. \tag{7.2}$$

From now on we will replace S_{UV} in our notation by S_E, to indicate that it represents the gravitational part of the action.

Indeed, we can use holography to identify the remaining part of the action S_{IR} with the quantum effective action of the low-energy gauge and matter theory

$$S_{\mathrm{IR}}(\phi, g) = \Gamma\left(\phi, \widehat{g}, a\right), \tag{7.3}$$

with couplings ϕ^I, in the background geometry defined by $\widehat{g}_{\mu\nu}$, and with a given IR cut-off scale set by a. In other words, instead of using the 5-d supergravity description all the way down into the IR region, we imagine that we can use the IR 4-d field theory to specify the dependence of Γ on the scale, metric and couplings.[4] Variations of Γ with respect to the latter two amount to insertions of the stress-energy tensor and operators $\mathcal{O}^I$, respectively. So in particular

$$\frac{1}{\sqrt{-g}}\,\frac{\delta\Gamma}{\delta g^{\mu\nu}} = \langle T_{\mu\nu}\rangle\,, \qquad \frac{1}{\sqrt{-g}}\,\frac{\delta\Gamma}{\delta\phi^I} = \langle\mathcal{O}_I\rangle\,. \tag{7.4}$$

From now on we will denote the low energy action $S_{\rm IR}$ by Γ.

Via these new identifications (7.1) and (7.3), the geometric split $S = S_{\rm UV} + S_{\rm IR}$ thus amounts to a division of the total effective action into a gravity and a matter contribution

$$S(\phi, g) = S_E(\phi, g) + \Gamma(\phi, g)\,. \tag{7.5}$$

Correspondingly, the on-shell configurations that extremize S are identified with solutions to the Einstein equations combined with the ϕ^I equations of motion

$$\frac{1}{2}\,U\,\widehat{g}_{\mu\nu} + \Phi\left(\widehat{R}_{\mu\nu} - \frac{1}{2}\,\widehat{R}\,\widehat{g}_{\mu\nu}\right) = \langle T_{\mu\nu}\rangle + T^{\phi}_{\mu\nu}\,, \tag{7.6}$$

$$\Box_{M}\phi_I + \frac{\partial U}{\partial\phi^I} + \frac{\partial\Phi}{\partial\phi^I}\,\widehat{R} = \langle\mathcal{O}_I\rangle\,, \tag{7.7}$$

where $T^{\phi}_{\mu\nu}$ denotes the stress-energy tensor of the ϕ-fields. Although these look like quite generic equations of motion, we know there must be a flow symmetry (6.1)–(6.2) that acts on its space of classical solutions. This symmetry can be used to show that e.g. the trace of the first equation, the equation of motion of the scale factor, can in fact be derived by combining the other equations. In addition, using the form (7.1) for $S_{\rm UV}$, one can show that (to leading order in the low energy expansion) this flow equation attains the exact form of the Callan-Symanzik equations for (the correlation functions derived from) the effective action Γ, including the correction term due the Weyl anomaly. We will not review this calculation here, but refer to [7].

8. RG dependence of the vacuum energy

The set of flow equations (6.1) through (6.4) can be used to deduce the classical RG trajectories of all relevant quantities as a function of the scale factor of the metric. To test the physics of these equations, we start with the simplest situation: in flat

[4] The suggestion here is that it might be possible in certain cases to *match* the field theoretic quantities (beta-functions and expectation values) locally with the corresponding supergravity quantities in a region where both sides are still under some control. One could imagine that in this way the two dual description can be "patched together" into one global description of a specific RG-flow, in which the supergravity provides an accurate description of the UV region and the 4-dimensional QFT of the IR regime.

space and with constant fields. To achieve this, we temporarily take the decoupling or decompactification limit in which the AdS-radius is sent to infinity. In this limit the 4-d inverse Newton constant Φ is infinite, so that we can consistently choose $\widehat{g}_{\mu\nu}$ to be flat.

So the only quantities we need to keep track of are the cosmological term U in S_E and the low energy effective action Γ. In addition to their explicit dependence on the scale a, both quantities also acquire an extra implicit a-dependence, induced by radial RG-flow. For constant fields, the only non-zero beta-functions are those of the couplings ϕ^I

$$a\,\frac{d\phi^I}{da} = \beta^I\,. \tag{8.1}$$

To extract the total scale dependence of both U and Γ, we again use the Hamilton-Jacobi equations

$$\gamma\left(a\,\frac{\partial}{\partial a} - \beta^I\,\frac{\partial}{\partial\phi^I}\right)U(\phi,a) \;=\; -2\,a^4\,V(\phi)\,, \tag{8.2}$$

$$\gamma\left(a\,\frac{\partial}{\partial a} - \beta^I\,\frac{\partial}{\partial\phi^I}\right)\Gamma(\phi,a) = 2\int\sqrt{-\widehat{g}}\,a^4\,V(\phi)\,. \tag{8.3}$$

The second of these equations is valid only on-shell. It tells us about the response of the low-energy matter theory under a small renormalization group step. We see that besides the RG adjustment (8.1) of the couplings, there's also an additional vacuum energy contribution, represented by the potential term on the right-hand side. As seen from the first eq. (8.2), however, the same contribution that is subtracted from Γ, is added in the same RG step to the potential term U. This suggests that U accumulates the vacuum energy contribution of all quantum fluctuations higher than the IR cut-off scale a. Notice that this cancellation between the vacuum energy contributions in the RG step is of course nothing particularly deep, since by definition S_E and Γ each represent complementary UV and IR parts of the total effective action S. Indeed, as shown earlier, S is on-shell invariant under (8.1).

Now let us take a first look at the cosmological constant Λ from this perspective. Λ is the total vacuum energy contained in U and Γ added together. Naively one might have expected that it will coincide with the infra-red limit of the vacuum energy contained in U, since it would seem reasonable to assume that, when we send the RG scale μ to zero, the contribution from Γ will vanish. It turns out, however, that this naive expectation is incorrect.

It is clear from their respective definitions, that the total action S and the high energy part S_E have rather different dynamical roles. Extrema of S have an immediate physical significance as classically stable field configurations, whereas S_E generally has non-zero variations that give us the RG-flow velocities. So the two actions can coincide in the far infra-red *only if* all couplings — including the RG-dependent metric $g_{\mu\nu}$ — would become static quantities under the RG-flow. It is not difficult to convince oneself, however, that our evolution equations in general predict that the couplings will continue to flow in the infra-red, and that as a result the IR contributions to the vacuum energy will remain substantial, even after the cut-off energy scale has been sent to zero. We thus conclude that there is an important numerical and physical distinction between the cosmological constant and the total RG accumulated vacuum energy contained in U.

9. The cosmological constant

Let us now return to the situation with dynamical 4-d gravity, but still assume that all fields are x^μ independent. We thus require that the metric $\widehat{g}_{\mu\nu}$ has a constant curvature $\widehat{R}$. Assuming that $\widehat{R}$ is small, we may expand the total action S to leading order as (the general case with $\widehat{R}$ arbitrary is treated in appendix A)

$$S\left(\phi, \widehat{g}, a\right) = \int \sqrt{-\widehat{g}} \left(F(\phi, a) + G(\phi, a)\widehat{R}\right). \tag{9.1}$$

Here G denotes the 4-d inverse Newton constant. The cosmological term F contains in addition to the RG-dependent vacuum energy U also contributions coming from the flow velocities of the couplings as well as from the holographic contraction rate of the warp factor. As we have just shown (and see also section 2 and appendix A), these extra contributions naturally adjust themselves to cancel the RG-variations in U. Indeed, as we will see momentarily, F is (on-shell) RG-invariant. As before, a parametrizes the dependence of both F and G on the fundamental string scale.

The equations of motion of ϕ^I and a read

$$\frac{\partial F}{\partial \phi^I} + \frac{\partial G}{\partial \phi^I}\widehat{R} = 0\,, \tag{9.2}$$

$$a\,\frac{\partial F}{\partial a} + a\,\frac{\partial G}{\partial a}\widehat{R} = 0\,. \tag{9.3}$$

In addition, the flow invariance (6.4) of S reduces to

$$\gamma\left(a\,\frac{\partial}{\partial a} - \beta^I\frac{\partial}{\partial \phi^I}\right) F = \{F, F\}\,, \tag{9.4}$$

$$\gamma\left(a\,\frac{\partial}{\partial a} - \beta^I\frac{\partial}{\partial \phi^I}\right) G = 2\{G, F\}\,, \tag{9.5}$$

where the bracket notation is defined in eq. (A.6). The consequences of this flow invariance are three-fold. First, it shows that not all equations in (9.2)–(9.3) are independent; e.g. once we have solved the ϕ^I equations of motion, the second equation for the scale factor a is automatically satisfied. A second consequence is that, once we have found some extremum (ϕ_0, a_0) of S at some given scale, we automatically obtain a whole 1-dimensional trajectory of critical points traced out by the RG-flow

$$a\,\frac{d\phi^I}{da} = \beta^I\,, \qquad \phi^I(a_0) = \phi^I_0\,. \tag{9.6}$$

Thirdly, all points on this trajectory have the same value for the cosmological constant $\widehat{R} = \Lambda$, as well as for F and G. Hence in particular, *if* the special solution in the UV happens to have zero cosmological constant Λ, *then* the flow symmetry will automatically imply the existence of stable flat space solutions that extend all the way down into the far IR.

We should mention that actions with a very similar flow symmetry have been the subject of active research in the past, precisely because they appear to offer a promising route towards eliminating the cosmological constant problem. These attempts were mostly abandoned, however, because there is also an important objection against this idea. This counter argument is most clearly explained in Weinberg's review article [1, section VI]. The main objection, as formulated in [1], is based on the argument that by general covariance, the cosmological term must necessarily be of the form

$$\int \sqrt{-\widehat{g}}\, a^4\, W(\phi)\,. \tag{9.7}$$

Since a cosmological term of this form is a monotonic function in a, it has no critical points whatsoever (except for the somewhat singular value $a = 0$), unless by chance or fine-tuning W happens to be zero. Thus our argument above leading to the existence of an RG trajectory (9.6) of critical points breaks down in this case. This is the reason why the presence of a vacuum energy term is usually associated with an obstruction to finding a flat space solution to the Einstein equation.

There are a number of important new ingredients in our set-up, however, that enable us to evade this "no-go theorem". First, it is quite clear that, because of the fact that a represents the RG scale, the cosmological term $F(\phi, a)$ in S can have a more general dependence on a than (9.7). This makes a crucial qualitative difference, since it opens the possibility that non-trivial critical points of F for which $a\,\frac{\partial}{\partial a}F = 0$ can exist. Moreover, as we have argued, there are good physical reasons to expect that the higher order dependence of F on a is precisely such that it allows for the existence of a whole line of critical points, describing a complete RG trajectory of solutions to (9.2).

Another important difference, relative to previous proposals in this direction, is that we identify this flow symmetry of S with the action of the renormalization group. Hence the fact that the equation of motion of S does not select a specific value for the scale factor a is not some unwanted instability of the system, but rather an essential ingredient needed for this identification. Indeed, it would be odd if the choice of RG scale would be determined dynamically, rather than by hand.

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Finally, it is important to notice that our results thus far do not point to any prefered value for Λ. Our main result, rather, is that we have given a new formulation of the RG and Einstein equations which is internally consistent with any value for Λ, and thus *including* the value $\Lambda = 0$. Given the fundamental clash between the RG intuition and the observational evidence of a small cosmological constant, we consider this a useful step forward.

10. Flat space stability

The central open question therefore, is whether it is possible to choose *natural* initial conditions in the far UV for which $\Lambda = 0$. Rather than attempting to give a conclusive answer this question, let us make some general remarks. Imagine we start with some stable supersymmetric compactification of 10-dimensional string theory. Eventually, this theory should want to break supersymmetry at lower scales. One could imagine two ways in which this may happen: either the planckian theory already contains soft non-supersymmetric terms, or there could be some dynamical mechanism that breaks supersymmetry via non-linear dynamics at some lower scale. In the first type of scenario, it seems quite clear that the planckian theory in effect already contains a non-zero cosmological term. The RG-stability of Λ will not help much in this case. In the dynamical scenario, on the other hand, it seems that this stability may become a very effective mechanism for keeping Λ small. Roughly, the large separation between the string and the supersymmetry breaking scale now translates via holography into a *physical* separation between the non-supersymmetric dynamics and the "Planck brane", that is, the place where the AdS-type geometry connects onto the compactification manifold $K_6 \times R^4$. The idea is that when this Planck brane is locally embedded in a true supersymmetric environment, it is protected from directly "feeling" the vacuum energy produced away from it in the bulk region. Hence in this case, it seems well possible that the 5-d supergravity equations allow solutions for which the 4-d world is (almost) flat.

To illustrate this point, consider as a basic — though rather special — example [15] the situation where the RG-flow of the low-energy theory is described by a non-supersymmetric domain wall separating two 5-d constant curvature regions, with different bulk cosmological constant V_+ and V_-. We imagine that for the field theory this means that its RG-flow connects a supersymmetric ultra-violet fixed point to a non-supersymmetric infra-red fixed point field theory. Hence $V_\pm = V(\phi_\pm)$ where ϕ_+ and ϕ_- denote the couplings of the UV and IR fixed point, respectively. Effective field theory tells us that this transition will produce a non-zero vacuum energy. The holographic manifestation of this vacuum energy is the intrinsic tension of the domain wall, which can be deduced by considering its contribution to the supergravity action S. For a flat domain wall, extended over a range of scales from a_+ to a_-, this becomes

$$\Delta S = \int \sqrt{-\widehat{g}}\,(U_+ - U_-) \tag{10.1}$$

with $U_\pm = U(\phi_\pm, a_\pm)$. The tension of the wall is obtained by taking the variation with respect the scale factor

$$\int \sqrt{-\widehat{g}}\,T = \frac{1}{4}\int \sqrt{-\widehat{g}}\left(a_+\frac{\partial U_+}{\partial a_+} - a_-\frac{\partial U_-}{\partial a_-}\right). \tag{10.2}$$

The right-hand side is precisely the difference between the external forces acting on the domain wall from the right and left, respectively, arising from its embedding in the bulk regions with curvature V_+ and V_-. Since by assumption $\partial_I U_\pm = 0$, the flat space Hamilton-Jacobi equation on each side reduces to

$$\frac{a_\pm}{4}\frac{\partial U_\pm}{\partial a_\pm} = a_\pm^4\sqrt{3V_\pm}\,. \tag{10.3}$$

By comparing with (10.2), this reproduces the special relation

$$T = a_+^4\sqrt{3V_\pm} - a_-^4\sqrt{3V_-} \tag{10.4}$$

between the tension and the bulk potential needed for the stability of the flat domain wall [3, 15]. Notice that this stability does not require any fine-tuning; instead the balance of force condition is automatically implied by the 5-d supergravity equations.[5] Evidently, the vacuum energy of the RG-transition in the boundary field theory, or tension in the domain wall, does not necessarily lead to a 4-d curvature.

To complete this example, we would also need to consider the region near the "Planck brane". Here one could argue that fine-tuning is needed. However, since this is the high energy region, we are allowed to use supersymmetry. There are many examples of consistent supersymmetric warped string compactifications that produce a flat 4-dimensional space-time. This shows that also for the Planck brane, the 10-d supergravity equations — combined with high energy supersymmetry — will automatically produce the required relation between the external force and internal tension to ensure stability. The point here is that the curvature of the 4-d geometry is determined by the local behavior of the fields in the Planck region. Hence the flat space stability will not be perturbed by the low energy supersymmetry breaking, *provided* the non-supersymmetric domain wall solution at lower scales is well enough localized so that it has a negligible effect on the supersymmetric solution near the Planck brane.

[5] Our reasoning here is in fact slightly circular, since to demonstrate the relation (10.4), we first needed to assume that a flat domain wall solution indeed exists. The point that's being made here, however, is that it does not require any fine-tuning of any couplings in the potential V to ensure stability of its possible domain wall solutions.

11. Discussion

In this paper we have presented a new angle on the cosmological constant problem, based on a combination of two-main ideas:

(i) the holographic correspondence between 4-d QFT and 5-d supergravity,

(ii) that via warped string compactifications of the RS-type this duality can be extended to 4-d field theories with gravity.

Our results show that in this type of scenario, there exists a natural dynamical adjustment mechanism that prevents the cosmological constant from being generated along the RG-flow, once it has been cancelled in the UV. This result appears to contradict the standard, and seemingly well founded, intuition about how Λ should behave. On the other hand, it is clear that any new proposal for dealing with this problem must involve a restriction on the applicability of the standard rules of 4-d effective field theory, as well as a sufficiently radical yet conservative modification of these rules.

The main difference between our set-up and the standard rule book is that the low energy equations of motion of 4-d gravity can now be derived only *after* solving the RG-flow equations induced by integrating out the matter. Just like in the RS-scenario, the massless 4-d gravity fluctuations correspond to zero-mode variations of the 5-geometry that look like $ds_5^2 = dr^2 + a(r)^2\widehat{g}_{\mu\nu}(x)$. The detailed warped shape of this RS graviton wave function depends critically on the stress-energy distribution, and therefore "knows" about the various phase transitions that occur at all lower-energy scales. This fact is clearly illustrated e.g. in the computation of the 4-d Newton constant outlined in appendix B, as well as in section 2. Our study shows that, by being able to adjust its form, the 4-d graviton is in effect able to make itself insensitive to the vacuum energy distribution spread out inside the 5-d warped geometry.

In spite of this new way of formulating gravitational dynamics, our set-up still seems consistent with most established rules of effective QFT. In particular, as outlined in section 8, we can identify a quantity U that behaves just like the vacuum energy induced by the matter fluctuations. However, this potential U *is not equal* to the cosmological term in the Einstein equations, also not in the far infra-red; it differs from it by terms arising from non-zero flow velocities of the couplings as well as from the contraction rate of the warp factor. These extra contributions naturally adjust themselves to cancel the RG-induced variations of the vacuum energy contained in U.

All our equations were derived using the 5-d supergravity approximation. It's indeed been one of our implicit assumptions that the rank of the high energy gauge group and gauge coupling are large enough to be in the right regime, at least initially. However, soon after the non-trivial RG behavior sets in, the 5-d solution will almost inevitably enter a strongly curved region in which the warp factor will approach zero within a finite proper distance. This behavior typically produces a naked singularity, near which our approximations certainly break down. Still we have reasons to hope that our main result, the RG stability of Λ, will continue to hold in this regime as well. One specific source of hope is that our equations are suggestive of a low energy approximation of the fundamental BV symmetry structure of closed string field theory. Via the dual correspondence between (planar) open string diagrams and closed string (tree) diagrams, one can indeed establish a direct interpretation of this BV symmetry as expressing an RG invariance of the effective action, induced by integrating out the open strings. Our suggestion is that this closed-open string duality may provide a fundamental explanation (that also extends to the weak coupling regime of the gauge theory) of much of the structure that we have found here [14].

The holographic interpretation of the warped geometry differs on a number of fundamental points with that of the original RS world-brane scenario [3]. In our approach, all physics taking place somewhere inside the 5-d bulk region is identified with physics happening inside our 4-d world. Conversely, 4-d phenomena that occur at different scales in our world, are represented as spatially separated in the extra direction. This means that there is no sharply localized world-brane anywhere within the warped space-time. In addition, there is no very clear distinction between Kaluza-Klein modes and other localized excitations of the 4-d field theory. It is therefore a subtle, but very interesting, question which possible new experimental signals one can typically associate with scenarios of this type.

Finally, a relatively serious short-coming of our discussion thus far is that we have only mentioned 5-d holography and not yet its 4-dimensional amplification, which should play an at least equally important role. It is indeed clear that 4-d holography will put even stronger restrictions on the validity of 4-d effective field theory than what we've discussed so far. Still, the incorporation of (at the moment somewhat better established) ideas from 5-d holography looks like a useful, albeit incomplete, step in the right direction.

Acknowledgments

This work is supported by NSF-grant 98-02484, a Pionier fellowship of NWO, and the Packard foundation. We would like to thank Christof Schmidhuber for useful discussions and collaboration in the early stages of this work. We also acknowledge helpful discussions with T. Banks, M. Berkooz, J. de Boer, S. Dimopoulos, S. Gubser, S. Kachru, I. Klebanov, A. Polyakov, L. Randall, E. Silverstein, R. Sundrum, and K. Skenderis.

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A. Radial dynamics for constant fields

In this appendix we summarize the Hamilton-Jacobi equations for the case of constant fields. Equations (2.2) and (2.3) are good starting points for analyzing the dynamics of a and ϕ^I; here however we will instead choose the set-up of sections 3 and 4, so that the analogy with the renormalization group equations remains somewhat more transparent. So let us again introduce the actions $S_{\text{UV}} = S_E$ and $S_{\text{IR}} = \Gamma$, and a total action $S = S_E + \Gamma$. Each of these actions now represent functions of just the couplings ϕ^I, the scale a and k:

$$S_E = S_E(\phi, a; k)\,, \qquad S = S(\phi, a; k)\,. \tag{A.1}$$

The Hamilton-Jacobi constraints now reduce to the following equations for the functions S_E and S:

$$\{S_E, S_E\} = a^8\, V + a^6\, k \tag{A.2}$$

and

$$\gamma \left(a\, \frac{\partial}{\partial a} - \beta^I \frac{\partial}{\partial \phi^I} \right) S = \{S, S\}\,. \tag{A.3}$$

Here we used the notations

$$\gamma = \frac{1}{24\, a^3}\, \frac{\partial S_E}{\partial a}\,, \tag{A.4}$$

$$\gamma\, \beta_I = \frac{1}{a^4}\, \frac{\partial S_E}{\partial \phi^I}\,, \tag{A.5}$$

$$\{S, S\} \equiv \frac{1}{48} \left(a\, \frac{\partial S}{\partial a} \right)^2 - \frac{1}{2} \left(\frac{\partial S}{\partial \phi^I} \right)^2 . \tag{A.6}$$

Note that (A.2) is equivalent to

$$\gamma^2 \left(1 - \frac{1}{24}\, \beta_I^2 \right) = \frac{1}{12} \left(V + \frac{k}{a^2} \right) . \tag{A.7}$$

The relation (A.2) defines a non-linear differential equation from which, with given initial conditions near the Planck region $a = a_0$, one can uniquely determine S_E — and thus also for γ and beta-functions β^I — as functions of ϕ, a and k. The detailed form of these functions of course critically depends on the form of the 5-d potential term V as well as on the specific choice of asymptotic conditions in the UV and IR. Once we know these functions, however, we can then use (A.3) to determine the function S at all scales from a given initial condition. Then finally, after we have determined S, we can distinguish the space of stable classical solutions (ϕ, a), by imposing the equations of motion

$$\frac{\partial S}{\partial \phi^I}(\phi, a; k) = 0\,, \qquad a\, \frac{\partial S}{\partial a}\, (\phi, a; k) = 0\,. \tag{A.8}$$

By virtue of eq. (A.3), the space of solutions to these equations of motion consists of 1-dimensional trajectories, generated by $d\phi^I/da = \beta^I$, where all solutions along this flow have the same value for the cosmological constant k.

B. 4-d Newton constant

In this second appendix we analyze the behavior of the Newton constant Φ in S_E under the RG-flow. This leads to an instructive comparison of our equations with those of the Randall-Sundrum scenario. We use the same notation as in appendix A.

Let us assume that we have obtained the β and γ as a function of a. Newton's constant can then be obtained as follows. First consider the quantity

$$\Phi(\phi, a) = \frac{\partial S_E}{\partial k}(\phi, a; k = 0) \tag{B.1}$$

which, recalling that $k = \widehat{R}$ represents the average curvature, can be seen to represent the coupling in front of the 4-d Einstein term in S_{UV} at the scale set by a. Using the equations (A.4) and (A.7) we derive that Φ satisfies the flow relation

$$\gamma \left(-a \frac{\partial}{\partial a} + \beta^I \frac{\partial}{\partial \phi^I} \right) \Phi = a^2 \,. \tag{B.2}$$

This equation determines the a-dependence Φ in terms of that of γ. Equation (B.2) can then be explicitly integrated to

$$\Phi(a) = \Phi(a_0) + \int_a^{a_0} \frac{a' \, da'}{\gamma(a')} \,. \tag{B.3}$$

The value of the integration constant $\Phi(a_0)$ is determined by the specific string compactification that describes the Planck scale physics.

It is useful to compare the above result (B.3) with the formula for the 4-d Newton constant in the Randall-Sundrum compactification scenario [3]. In the set-up of [3], the 4-d Newton constant is expressed via a similar integral as (B.3) with $\Phi(a_0) = 0$ and with $\gamma(a) = \text{const}$. The location a_0 in their case corresponds to the location of the "Planck brane" that cuts off the asymptotic AdS-region. The above expression, however, also applies to more general ways of cutting off the AdS-space by means of the warped string compactifications described in section 2. In this case, a_0 could be chosen to represent the place where the AdS-space is glued into the compactification geometry $R^4 \times K^6$. The integration constant $\Phi(a_0)$ will then be a finite number, determined by the 10-d Newton constant κ_{10} and the volume of the K_6 with the six-ball B_6 removed, via

$$\Phi(a_0) = \frac{V_6'}{\kappa_{10}} \,, \qquad V_6' = \text{Vol}\,(K_6 - B_6) \,. \tag{B.4}$$

This relation directly follows from the expression (4.3) for S_{UV}. Notice that the total a_0-dependence exactly cancels in the formula for $\Phi(a)$, since small shifts in the endpoint of the a-integration are compensated by small shifts in radius of the six-ball B_6 inside K_6. This cancellation in ensured by the relation between the 5-d and 10-d Newton constant $1/\kappa_5 = V_5/\kappa_{10}$ with $V_5 = \mathrm{Vol}(K_5)$.

The true 4-d Newton constant κ, as seen at long distances, is the IR fixed point value of $1/\Phi$. Hence

$$\frac{1}{\kappa} = \frac{V_6'}{\kappa_{10}} + \int_0^{a_0} \frac{a\, da}{\gamma(a)}\,. \tag{B.5}$$

This is a finite result, because $\gamma(a)$ will always remain non zero, and usually will even grow very large, in the IR region $a \to 0$. It thus seems very plausible that (B.5) indeed expresses the physical Newton constant of the low energy action S.

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